# Histograms Analysis of the Microstructure of Halftone Images 

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#### Abstract

Three quality metrics commonly measured in the microdensitometric analysis of halftone images are the halftone dot area fraction, F , the mean reflectance of the paper between the halftone dots, $\mathrm{R}_{\mathrm{p}}$, and the mean reflectance of the dots, $\mathrm{R}_{\mathrm{i}}$. These metrics are commonly measured from the histogram of the image captured through microscope optics. However, local variations (noise) in the image, the physical spread of the edges of halftone dots, and the scatter of light in the image often lead to difficulties in estimating $R_{i}, R_{p}$, and $F$ from the histogram. In order to improve histogram analysis of the microstructure and image quality of halftone images, a model for the behavior of histogram curves has been developed. This model can be fit to the experimental histograms by a minimum RMS deviation in order to estimate $R_{i}, R_{p}$, and $F$ and other quality metrics of the histogram image.


## Introduction

The quality of printed halftone images is governed in large part by the spatial distribution of gray levels within the halftone structure. Both the lateral distribution of ink and the optical scattering of light in the printed paper contribute to a phenomenon often called "dot gain" in which the halftone image absorbs more light than anticipated for a perfect bi-modal image of ink or no-ink. These effects can be characterized quantitatively with image microdensitometery in which a digital image of the halftone pattern is captured through microscope optics and subjected to various types of image analysis. Three particularly useful metrics extracted from this kind of analysis are the reflectance, $\mathrm{R}_{\mathrm{i}}$, of the ink dots, the reflectance, $\mathrm{R}_{\mathrm{p}}$, of the paper between the dots, and the halftone dot area fraction, F. These measured metrics add up to the mean reflectance of the overall halftone image as $\mathrm{R}=\mathrm{F} \mathrm{R}_{\mathrm{i}}+(1-\mathrm{F}) \mathrm{R}_{\mathrm{p}}$.

The values of $R_{p}, R_{i}$, and $F$ can be estimated by analyzing the histogram of the image captured through the microscope. The histogram is a plot of the frequency of occurrence of gray levels of reflectance ( $0 \leq \mathrm{R} \leq 1$ ) as a function of the value of R. Figure 1 illustrates a typical histogram from a microdensitometry image of a 65 LPI halftone (AM clustered dot) printed by a traditional offset lithography. The microdensitometry image covered a field of view of 5 mm , and the histogram clearly shows two gray level populations; one for the ink and one for the paper between the dots. Figure 1 also clearly illustrates how the values of $R_{p}, R_{i}$, and $F$ are estimated from the histogram.

Visual inspection of the histogram leads to a satisfactory estimate of these three parameters in this illustration. However, this is not always the case, and the object of the current work was to develop improved methods for histogram analysis as part of a study of the optical and physical behavior of digital halftones produced by nonimpact printing devices.


Figure 1: Histogram of 65 LPI AM halftone printed by offset lithography, measured at 5 mm field of view (FOV)

## Histogram Characteristics

Nominally, image histograms carry no spatial information. They are only probability density functions for the occurrence of reflectance levels in the image, regardless of where in the image the gray level occurs. However, by knowing a priori that the image represents a halftone, we can relate properties of the histogram to spatial properties of the halftone. The simplest example is the use of the threshold value of $\mathrm{R}_{\mathrm{t}}$, as illustrated in Figure 1, to estimate the area coverage of the halftone dots, $F$. If the halftone dots were perfectly formed, then the histogram would be truly bimodal, with a delta function of area F and another of area 1F for the ink and paper area fractions, respectively. However, due both to noise and to the spread or blurring of the halftone dots, the populations around $\mathrm{R}_{\mathrm{i}}$ and $\mathrm{R}_{\mathrm{p}}$ are spread out. $\mathrm{R}_{\mathrm{t}}$ represents the reflectance at the boundary between dot and paper, defined as the point of steepest slope between the dot and the paper, as illustrated schematically in Figure 2.

## R



Position, x in mm
Figure 2: Illustration of a line scan across halftone dots.

Well formed halftone dots such as those represented in Figure 1 are not always observed. Rather, histograms such as those shown in Figures 3 and 4, representing ink jet error diffusion images at 300 dpi at approximately $\mathrm{F}=0.5$ and F $=0.1$ are quite common. While one might still segment the histogram of Figure 3 repeatably, one has less confidence in the physical significance of the results, and Figure 4 offers a serious problem in selecting an objective and repeatable value of $\mathrm{R}_{\mathrm{t}}$. Nevertheless, if one observes the image of the halftone dots corresponding to the histograms in Figures 3 and 4 , halftone dots are easily observed and it is easy in both cases to see the boundary between dot and paper. This is a not uncommon problem with histogram segmentation. The eye is an excellent detector of boundaries.


Figure 3. Histograms at 5 mm FOV of error diffusion dot pattern printed by thermal ink jet at 300 dpi at a nominal $F=0.5$.


Figure 4. Histograms at 5mm FOV of error diffusion dot pattern printed by thermal ink jet at 300 dpi at a nominal $F=0.05$.

$$
\begin{equation*}
\mathrm{R}=\frac{\mathrm{R} \max -\mathrm{R} \min }{1+\exp [-\mathrm{a}(\mathrm{x}-\mathrm{b})]}+\mathrm{R} \min \tag{1}
\end{equation*}
$$

## Modeling the Bimodal Histogram

In order to improve histogram analysis and segmentation to estimate the $R_{p}, R_{i}$, and $F$ parameters, a model of histogram behavior for bi-modal images was developed. The basis of the histogram is a description of the edge trace of the dot, such as shown in Figure 2. The edge model chosen was a simple, symmetrical sigmoidal function ${ }^{2}$ shown in equation (1) and illustrated in Figure 5 for $\operatorname{Rmin}=0.3, \operatorname{Rmax}=0.7, \mathrm{a}=10$, and $\mathrm{b}=0.5$. This represents an average, one dimensional, equivalent edge from the middle of the paper $(x=0)$ to the middle of a dot
$(x=1)$. The value of "a" is an index of the steepness of the edge of the dot and is proportional to the slope of the curve halfway between Rmin and Rmax. The b term represents the lateral position of the curve such that the mid point between Rmin and Rmax occurs at a value of $x=b$. The values of Rmax, Rmin, $b$, and a in the model may be used as objective estimates of $R_{p}, R_{i}, F$, and the sharpness of the edge of the dots. For a real, two dimensional $R(x, y)$ halftone dot this equivalent edge is not the same as an actual scan of the edge of a dot. However, this one dimensional equivalent edge provides a basis for modeling the shape of the halftone histogram as follows.


Figure 5: The edge modeled by equation (1) with Rmin $=0.3$, Rmax $=0.7, a=10$, and $b=0.5$.

The histogram represents the frequency of occurrence of a given value of $R$. This frequency is inversely proportional to the slope of the $R$ versus $x$ curve. Slopes of zero have a high occurrence, for example, and the mid-point of the dot where the slope is highest has the lowest frequency of occurrence. Thus, the histogram of a halftone dot should be represented by equation (2).

$$
\begin{equation*}
\mathrm{H}(\mathrm{R})=\left[\frac{\mathrm{dR}}{\mathrm{dx}}\right]^{-1} \tag{2}
\end{equation*}
$$

Figure 6 illustrates the histogram corresponding to equation (2) for the curve of Figure 5.


Figure 6: The histogram modeled by equation (2) for the edge in Figure (5).

The curve in Figure 6 looks only somewhat like an experimental histogram. In order to further model the behavior of actual, experimental histograms one has to recognize that there is a variation among halftone dots as well as a variation, or granularity, in the paper reflectance. This noise effect can be added to the model by defining an average noise metric, $S(x)$, as a gaussian probability density function.

$$
\begin{equation*}
S(R)=\frac{1}{\sigma \sqrt{2 \pi}} \exp \left[\frac{(\mathrm{R}-0.5)^{2}}{2 \sigma^{2}}\right] \tag{3}
\end{equation*}
$$

The functions H and S were convolved by taking the Fourier transform of the functions, multiplying, and then the inverse Fourier transform. As an example, the histogram of Figure 6 was convolved with equation (3) at $\sigma=0.02$ in units of R. The result is illustrated in Figure 7 and looks much more like an experimental histogram.


Figure 7: The histogram of Figure 6 with noise added by equation (3) at $\sigma=0.02$ in units of $R$.

## Application of the Model

Figures 8 and 9 illustrate the use of this histogram model. Both show the histogram data of Figures 3 and 4 as dots and the model as a solid line. The fit represents a minimum RMS deviation between the line and the data, and both examples show a reasonably close agreement between data an model. Thus, statistically fitting the model to the data does provide a repeatable and objective method for estimating the parameters $\mathrm{R}_{\mathrm{p}}=\mathrm{Rmax}, \mathrm{R}_{\mathrm{i}}=\mathrm{Rmin}$, and $\mathrm{b}=\mathrm{F}$. However, inspection of the results for Figure 8 show a somewhat surprising result. The value of Rmax which best fits the data is significantly higher than any actual reflectance in the image. Moreover, the estimated fit value of $b$ is essentially unity. The reason can be seen in Figure 10 for the equivalent edge which produced the modeled histogram. The results illustrate that Rmax and Rmin are not the same as the mean values $R_{p}$ and $R_{i}$. In addition $b$ is not exactly equivalent to F . Thus, while the model provides a good fit to the data, some independent definitions for the mean values of these quality metrics must be defined carefully in order to interpret the results of the model fit.

An alternative application to histogram analysis is to apply this model in reverse. By integrating the experimental
histogram function, one can generate a one dimensional "equivalent edge" for the image. The ersatz spatial dimension, $x$, is defined in units of area fraction, $0<x<1$, as follows.

$$
\begin{equation*}
x(R)=\int_{0}^{R} H(R) / \int_{0}^{1} H(R) \tag{4}
\end{equation*}
$$



Figure 8: The data of Figure 3 fit to the model with parameters Rmin $=0.266$, Rmax $=0.496, a=11.0$, and $b=0.53$.


Figure 9: The data of Figure 4 fit to the model with parameters Rmin $=0.243, \operatorname{Rmax}=0.442, a=8.4$, and $b=0.999$.


Figure 10: The edge modeled by equation (2) for the minimum RMS deviation fit in Figure 9.


Figure 11: Reflectance as a funcon of the integral function of the histogram data in Figure 3.

Figure 11 illustrates the calculation of the equivalent edge for the halftone image represented by the histogram in Figure 3. It should be noted that spatial information is somewhat confused in this analysis, and the slope of the equivalent edge trace in Figure 11 is in part a function of the random noise in the original image as well as the physical sharpness of the actual dot edges. Nevertheless, calculation of this equivalent edge does provide an objective evaluation of the edge quality averaged over the entire halftone image.

## References

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